MYP Y10

Logarithms

Background

So far, we have been able to solve power equations with a mixture of memory work, the ability to use the rules of indices and graphs.

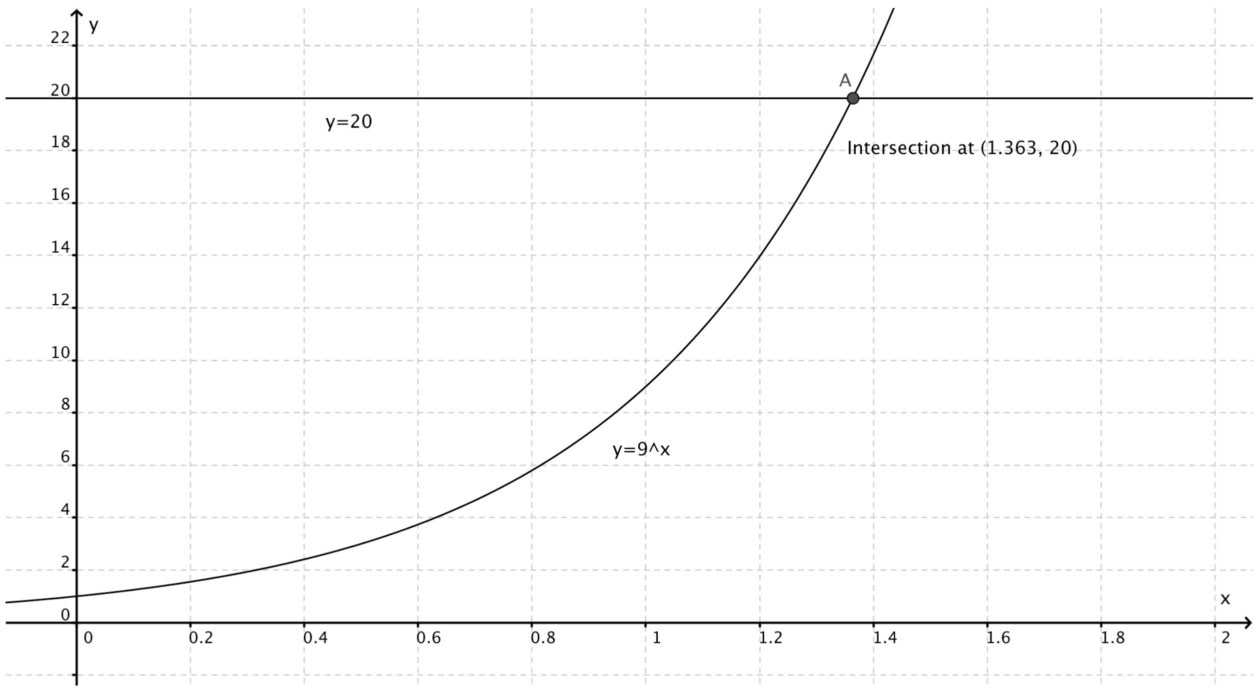
For example:

Solving $3^x = 27$ is quite easy because we remember $3^3 = 27$ (so $x = 3$)

When we change the equation to $9^x = 27$, it's not too difficult because we recognise that 9 and 27 are both powers of 3. So we re-write like:

$$9^x = 27 \quad \Rightarrow \quad (3^2)^x = 3^3 \quad \Rightarrow \quad 3^{2x} = 3^3 \quad \Rightarrow \quad 2x = 3 \quad \Rightarrow \quad x = 1.5$$

When we have an equation like $9^x = 20$, we cannot use either of the above methods. We can use technology, or graphs (or a mixture of both). A GDC will be able to solve the equation for us. Or we could draw a graph of $y = 9^x$ and see where the graph cuts the line $y = 20$, like in the Geogebra screenshot below:



The solution to the equation $9^x = 20$ is $x = 1.36$ approximately

There is another method – using the algebra of logarithms.

Logarithms give us another way of writing an exponential equation.

In short, $\boxed{\log_a b = c \quad \text{is another way of writing} \quad a^c = b}$ a is called the **base** of the logarithm

Let’s try with some simple examples:

$\log_{10}100 = 2$ is another way of writing $10^2 = 100$

$\log_4 64 = 3$ because $4^3 = 64$

$\log_5 625 = 4$ because $5^4 = 625$

$\log_2(1/4) = -2$ because $2^{-2} = 1/4$

Calculators are generally programmed with two types of base for their logarithms – 10 and e (a very special number you will learn about in the Diploma Programme).

Log to the base 10 should be written $\log_{10}$, but often it is just written as log.

So when you find the $\log_{10}$ of a number, you’re really finding the power that 10 should be raised to in order to equal the number.

In other words, $\log_{10}60$ can be found from your calculator. Call it x, for now. In calculating $\log_{10}60$, you’re really solving $10^x = 60$.

A. Find the following (without using technology):

(a) $\log_2 4$ **(b)** $\log_5 25$ **(c)** $\log_2 64$ **(d)** $\log_{10}10000$ **(e)** $\log_6 216$ **(f)** $\log_{87}1$

B. Find the following (again without using technology):

(a) $\log_2 0.5$ **(b)** $\log_5 0.2$ **(c)** $\log_4 0.25$ **(d)** $\log_{10}0.01$

C. Solve the following:

(a) $10^x = 45$ **(b)** $10^x = 450$ **(c)** $10^x = 2000$ **(d)** $10^x = 0.1$

D. Find the following (again without using technology):

(a) $\log_4 32$ **(b)** $\log_9 27$ **(c)** $\log_8 32$ **(d)** $\log_{25}125$

E. Find:

(a) $\mathrm{Log}_{10}1$ **(b)** $\log_{10}10$ **(c)** $\log_{10}10^2$ **(d)** $\log_{10}10^n$

F. Find:

(b) $\log_a 1$ **(b)** $\log_a a$ **(c)** $\log_a a^2$ **(d)** $\log_a a^n$